ДЗ 22.11.12

Пресметнете

$$\lim \frac{2^n+3^n}{4^n} = \lim \frac{2^n}{4^n} + \frac{3^n}{4^n} = \lim \left(\frac{2}{4}\right)^n + \left(\frac{3}{4}\right)^n =$$

$$0+0=0$$

$$q^n = \begin{cases} 0 & |q|<1 \\ \infty & |q|>1 \end{cases}$$

$$\lim \frac{2^{n+1}+3^{n+1}}{2^n+3^n} = \lim \frac{3^n\left(2\left(\frac{2}{3}\right)^n+3\right)}{3^n\left[\left(2/3\right)^n+1\right)} =$$

$$\frac{2\left(\frac{2}{3}\right)^n+3}{(2/3)^n+1} = \frac{3}{1} = 3$$

$\lim \frac{a^n}{1+a^n}$ където $a \neq -1$, $a \in \mathbb{R}$

Нека $|a|<1$

тогава $a^n \to 0 \Rightarrow \lim \frac{a^n}{1+a^n} \to 0$

при $|a|>1 \Rightarrow$

$a^n \to \infty \Rightarrow \lim \frac{a^n}{a^n(1+a^{-n})} = \frac{1}{1+0} = 1$

$$a^{-n} = 1/a^n = \left(\frac{1}{a}\right)^n \to 0$$

$\lim (1+i)^n = ?$

$$|1+i| = \sqrt{(1+i)(1+i)} = \sqrt{(1+i)(1-i)} = \sqrt{1-i^2} = \sqrt{1+1} = \sqrt{2}$$

$$\sqrt{2} > 1 \Rightarrow \lim (1+i)^n \to \infty$$

Сим $\frac{1}{n}\left(\frac{1+i\sqrt{3}}{2}\right)^n$

при комплексно число се гледа модул Сигнали

$$\left|\frac{1}{n}\left(\frac{1+i\sqrt{3}}{2}\right)^n\right| = \frac{1}{n}\left|\frac{1+i\sqrt{3}}{2}\right|^n \Rightarrow \frac{1}{2}+\frac{i\sqrt{3}}{2} =$$

$$= \sqrt{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)} = \sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{i\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1+3}{4}} = 1$$

$$\frac{1}{n}\cdot 1 = \frac{1}{n} \longrightarrow 0$$

$\lim\limits_{n\to\infty} \frac{n^k}{a^n}$ при $a>1$ $a\in\mathbb{R}$

$\exists\, \varepsilon>0$ т.ка $a = 1+\varepsilon \Rightarrow$

$$a^n = (1+\varepsilon)^n = \sum_{k=0}^{n} \binom{n}{k}\varepsilon^k$$

$$n^k a^n = \begin{cases} 0 & |a|<1 \\ \infty & |a|\geq 1 \end{cases}$$

$$\lim \frac{n^2(2i)^n - 3^n}{3^n + n^2} = \lim \frac{3^n\left(n^2(2/3\, i)^n - 1\right)}{3^n\left[1 + n^2\left(\frac{1}{3}\right)^n\right]} =$$

$$\frac{-1}{1} = -1 \qquad |2/3\, i| = \frac{2}{3} < 1$$

$$\lim \frac{n^3 4^n + n^2 3^n + (-5)^n}{n^5 2^n - n 3^n + 5^n} =$$

$$\lim \frac{5^n \left(n^3 (4/5)^n + n^2 (3/5)^n + (-1)^n \right)}{5^n \left[n^5 (2/5)^n - n (3/5)^n + 1 \right]} \Rightarrow$$

неограничена и несъществува
(от $(-1)^n$) има подредица $\to 1$ и
подредица $\to -1$

$a_{2n-1} \to -1$, $a_{2n} \to 1$ $\lim \nexists$

Сметнете $\lim a_n$

$$a_n = i^n$$
$$|i| = \sqrt{i\bar{i}} = \sqrt{-1^2} = 1 \Rightarrow a_n = 1$$

$$\left.\begin{aligned} a_{4k-3} &= i \\ a_{4k-2} &= -1 \\ a_{4k-1} &= -i \\ a_{4k} &= 1 \end{aligned}\right\} \Rightarrow \lim a_n \nexists$$ но има 4 граници на сгъстяване $i, -1, -i, 1$

Да се намери дефиниционната област на

$\lim a_n$, където $a_n = \frac{a^n}{n!}$. Ще докажем че a_n е монотонна и ограничена $\Rightarrow$ (сх от Th)

$\frac{a_{n+1}}{a_n} \Rightarrow$ a_n - расте ако $a_n < a_{n+1}$
a_n - намалява ако $a_n > a_{n+1}$

Деф

a_n е монотонна ако расте или намалява

$$\frac{a_{n+1}}{a_n} \geq \frac{a^{n+1} n!}{(n+1)! a^n} = \frac{a}{n+1} < 1 \quad \text{за } n > a-1$$

$$a_{n+1} < a_n$$

$$\lim \frac{3^{-n} n! + 5^n}{n!} i = \lim 3^{-n} i + \frac{5^n}{n!} i = 0$$

$$|3^{-n} i| = \left|\frac{i}{3^n}\right| = \frac{1}{3^n} \longrightarrow 0$$

$$\left|\frac{5^n}{n!} i\right| = \frac{5^n}{n!} \longrightarrow 0$$

$$\lim \sqrt{\frac{1+3^n}{n5^n}} \Rightarrow a_n = \frac{1+3^n}{n5^n}$$

$$\lim a_n = \lim \frac{1+3^n}{n5^n} = \frac{1}{n}\frac{1}{5^n} + \frac{1}{n}\frac{3^n}{5^n} \Rightarrow \lim a_n \to 0$$

$$\Rightarrow \lim \sqrt[n]{\frac{1+3^n}{n5^n}} = \lim \sqrt{\frac{3^n(1+3^{-n})}{n5^n}} = \lim \sqrt[n]{\left(\frac{3}{5}\right)^n \frac{1}{n}\left(1+3^{-n}\right)} =$$

$$\frac{3}{5} \frac{1}{\sqrt[n]{n}} \sqrt[n]{1+(1/3)^n} \Rightarrow 3/5 \cdot 1 \cdot 1 = 3/5$$

$$\lim \sqrt[3]{n^4}\left(\sqrt[3]{n^2+1} - \sqrt[3]{n^2-1}\right) \longrightarrow 2/3$$

Сметнете

$$\lim_{n\to\infty} \frac{\sqrt[3]{1+a_n} - 1}{a_n} \qquad \text{ако } a_n \to 0 \text{ за } \forall n \in \mathbb{N}$$

$$\lim \frac{1 + a_n - 1}{a_n\left(\sqrt[3]{1+a_n}^2 + \sqrt[3]{1+a_n} + 1\right)} = \frac{1}{3}$$